Identities involving the biexponential derivative functions

Biexponential component functions

Definitions of the biexponential component functions

The biexponential component functions were defined previously [1] as follows.

$$(u,v)^{(n,m)} \stackrel{\text{def}}{=} \frac{u^n}{n!} \cdot \frac{v^m}{m!}$$

and

$$(u;v)^n \stackrel{\text{def}}{=} (u,v)^{(n,n)}.$$

Identities involving the biexponential component functions

I will refer to $(u;v)^n$as the symmetric biexponential component functions because

$$(u;v)^n = (v;u)^n$$

for every natural number *n*. Similarly, I will refer to $(u,v)^{(n,m)}$as the asymmetric biexponential component functions because, in general $(u,v)^{(n,m)} \neq (u,v)^{(m,n)}$. Instead, we have the following identity.

$$(u,v)^{(m,n)} = (v,u)^{(n,m)}$$

Biexponential derivative functions

Definitions of the biexponential derivative functions

As before the biexponential function is defined as

$$0^{th}(u,v) \stackrel{\text{def}}{=} e^{(u;v)} \stackrel{\text{def}}{=} (u;v)^0 + (u;v)^1 + (u;v)^2 + (u;v)^3 + \cdots$$

and (as before) the positive biexponential derivative functions are defined for every positive integer *n by*

$$n^{th}(u,v) \stackrel{\text{def}}{=} (u,v)^{(0,n)} + (u,v)^{(1,n+1)} + (u,v)^{(2,n+2)} + (u,v)^{(3,n+3)} + (u,v)^{(4,n+4)} + \ \dots .$$

To simplify the identities involving the biexponential derivative functions, I will extend their definition to negative biexponential derivative functions as follows.

$$(-n)^{th}(u,v) \stackrel{\text{def}}{=} (u,v)^{(n,0)} + (u,v)^{(n+1,1)} + (u,v)^{(n+2,2)} + (u,v)^{(n+3,3)} + (u,v)^{(n+4,4)} + \ \dots .$$

Identities involving the biexponential derivative functions

Employing the extended definition of the biexponential derivative functions we have the following identities. For every integer *n*,

$$n^{th}(u,v) = (-n)^{th}(v,u),$$

$$\frac{\partial}{\partial u} n^{th}(u,v) = (n+1)^{th}(u,v),$$

$$\frac{\partial}{\partial v} n^{th}(u,v) = (n-1)^{th}(u,v)$$

and, in particular,

$$\frac{\partial}{\partial u}\frac{\partial}{\partial v} n^{th}(u,v) = n^{th}(u,v).$$

Proposition (the biexponential derivative proposition)

Given the above identities, I propose that the biexponential derivative functions are related to the exponential according to the following identity.

$$e^{u+v} = \sum_{n=-\infty}^{\infty} n^{th}(u,v)$$

tags

biexponential function; biexponential derivative functions;
biexponential component functions; component functions;
symmetric biexponential component functions; symmetric component functions;
asymmetric biexponential component functions; asymmetric component functions;
positive biexponential derivative functions; negative biexponential derivative functions;
biexponential derivative proposition